DEMODULATORS

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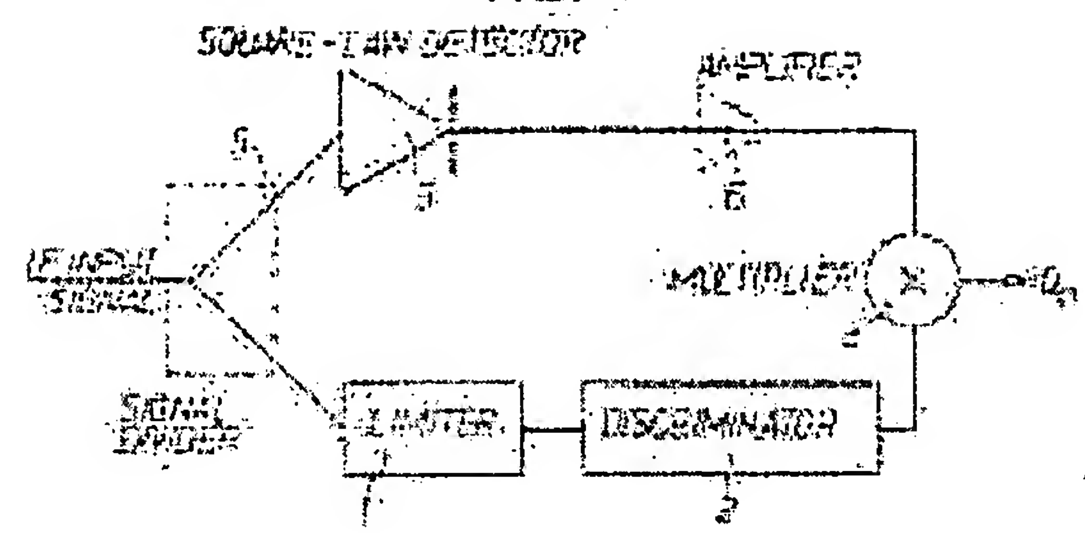
Abstract of **GB 1351939 (A)**

1351939 Frequency demodulators PLESSEY CO Ltd 7 July 1971 [8 July 1970] 33238/70 Heading H3A An FM demodulator comprises a frequency discriminator 2, an amplitude detector 3, the outputs of 2, 3 being multiplied together at 4 to obtain the required output signal. The amplitude detector 3 may be a square law detector or a linear (envelope) detector. The input signal may be applied via a signal divider 5 with a limiter 1, and amplifier 6 being included in the respective paths. The above arrangement is especially useful for the demodulation of FM signals with a low S/N ratio with the threshold performance of the discriminator 2 being enhanced by correlation techniques involving the use of the amplitude detector 3 and the multiplier 4.

FIG. 1



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(19)

(54) IMPROVEMENTS IN OR RELATING TO DEMODULATORS

(71) We, THE PLESSEY COMPANY LIMITED, a British Company of 2/60 Vicarage Lane, Ilford, Essex, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to demodulators and relates more specifically to F.M. (frequency modulation) demodulators. It is particularly concerned with the demodulation of frequency modulated signals which contain a high degree of adding interfering noise, such that a normal demodulator would suffer a sharp degradation of performance due to the well known "threshold effect".

According to the present invention there is provided a F.M. demodulator comprising frequency discriminator means for detecting an input frequency modulated signal and amplitude detector means for detecting the amplitude of said signal, the output of the frequency discriminator means and amplitude detector means being multiplied together to afford the required output signal.

The effect of the multiplication is to significantly reduce the so-called clicks, (arising from the added noise which is the basic cause of the threshold phenomenon), as will hreeinafter become apparent.

In one arrangement according to the invention the amplitude detector means may take the form of a square-law detector and in another arrangement the amplitude detector means may take the form of a linear (envelope) detector.

In carrying out the invention the input signal may be fed to signal dividing means which affords signals for the amplitude detector means and the frequency discriminator means and conveniently limiter means may be provided in the input of the frequency discriminator means.

It may also be found necessary to provide amplifier means for amplifying the output of the amplitude detector means.

The foregoing and other features of the invention will now be described and a theoretical analysis given, reference being made to the accompanying drawings, in which

Figure 1 is a block schematic diagram of a F.M. demodulator according to the present invention, and

Figures 2 and 3 show in graphical form comparisons of the demodulator of Figure 1 with that of a conventional frequency modulation feedback receiver—as referred to in the theoretical analysis hereinafter provided.

In the F.M. demodulator shown in Figure 1 of the drawings, an input frequency modulated signal, conveniently derived from the intermediate frequency (I.F.) stage of an F.M. receiver is fed to conventional F.M. discriminator stage consisting of a limiter 1 and a frequency discriminator 2 and also to a square-law detector 3. The output from the discriminator 2 and detector 3 are then linearly multiplied together in a multiplier 4 which affords a demodulated output signal D_o. In practice it may be found necessary to provide a signal divider circuit 5 at the inputs of the limiter 1 and detector 3 and an amplifier 6 may also be required in the output of the square-law detector 3. The signal divider 5 and amplifier 6 are shown in broken lines in Figure 1. As will hereinafter be explained, in some applications it may be possible to substitute a linear e.g. envelope detector for the square-law detector 3.

The demodulator of Figure 1 was evolved from a consideration of possible demodulator designs based upon the high correlation between dips in the envelope of the I.F. input signal and clicks appearing in the output of a conventional F.M. receiver, and has led to a new method of demodulation which is simpler to implement than the schemes already proposed by Hess (Ref. 1), Calandrino and Immovili (Refs. 2, 3), and should give better performance at low carrier-to-noise ratios.

The operation of the F.M. demodulator of Figure 1 is found to be amenable to a quite rigorous analysis and has the attractive

[*Price 25p*]

feature that the manner in which changes in the modulation parameters affect the performance can be easily determined.

On first sight, since a limiter is specifically included to remove amplitude fluctuations it would appear that the envelope can play no part in the demodulation process once limiting has been effected. A closer examination of a conventional F.M. receiver output reveals however, that the square of the envelope appears as the denominator in the expression for the discriminator output. In familiar notation (the reader is here referred to the list of symbols), this is illustrated by the following:—

The combination of modulated carrier and flat thermal noise at the demodulator input is:—

$$E\cos[\omega_o t+\mu_t]+\sum_{-\omega_B}^{\omega_B} b\cos[(\omega_o t+qt)+\alpha_q] \qquad \ldots.(1)$$

or equally,

$$E\cos[\omega_o t+\mu_t]+I_c(t)\cos\omega_o t-I_s(t)\sin\omega_o t \qquad \ldots.(2)$$

Here $I_c(t)$ and $I_s(t)$ are the inphase and quadrature components of the noise (Ref. 4). In terms of summations of pseudo-randomly phased tones consistent with the form used in (1) to represent the thermal noise, they are given by:—

$$I_c(t)=\sum_{-\omega_B}^{\omega_B} b\cos(qt+\alpha_q)$$

$$I_s(t)=\sum_{-\omega_B}^{\omega_B} b\sin(qt+\alpha_q)$$

It may be mentioned that $I_c(t)$ and $I_s(t)$ are dependent Gaussian variables but they are uncorrelated. Also the mean square values of I_c and I_s are the same and equal the total noise power.

The envelope of the carrier+noise sum is:—

$$V(t)=\{[E\cos\mu_t+I_c(t)]^2+[E\sin\mu_t+I_s(t)]^2\}^{\frac{1}{2}} \qquad \ldots.(3)$$

$$=\{P(t)^2+Q(t)^2\}^{\frac{1}{2}} \qquad \ldots.(4)$$

The definitions of P and Q being obvious.

The output voltage from the limiter-discriminator combination is then given by the well-known form:—

$$v_o(t)=\frac{P\dot{Q}-Q\dot{P}}{(P^2+Q^2)}=\frac{P\dot{Q}-Q\dot{P}}{V^2} \qquad \ldots.(5)$$

where the dots shown over Q and P indicate differentiation.

It is thus found that V^2 appears in the denominator. Now since a spike or click is likely to be present in $v_o(t)$ if V becomes small (the particular conditions for a click are $Q=0$, $P<0$, $\dot{Q}$ +ve or −ve) it is suggested that the click will be significantly reduced in level if v_o is multiplied by V^2 or the normalised quantity

$$\left(\frac{V}{E}\right)^2.$$

Thus in the arrangement of Figure 1 it is seen that the incoming signal is split into two parts one part being fed to a conventional F.M. discriminator stage and the other to a square-law detector. The outputs are then multiplied so that the signal accepted finally may be written as:—

$$v(t)=\frac{P\dot{Q}-Q\dot{P}}{E^2} \qquad \ldots.(6)$$

The next section presents a description of the analysis necessary to determine the power spectrum of this signal when the modulation on the input-carrier is modelled by a band of Gaussian noise.

Theoretical Analysis

Using Exp (3), straightforward substitution into Exp (6) gives:—

$$v(t)=M_t+\frac{\dot{I}_s}{E}\cos\mu_t-\frac{\dot{I}_c}{E}\sin\mu_t+M_t\left(\frac{I_c}{E}\cos\mu_t+\frac{I_s}{E}\sin\mu_t\right)-\left(\frac{I_s\dot{I}_c}{E^2}-\frac{I_c\dot{I}_s}{E^2}\right) \quad \ldots(7)$$

$$=M_t+A_t-B_t \quad \ldots(8)$$

Here

$$M_t=\frac{d\mu_t}{dt}$$

and is the desired signal on demodulation.

Since no time varying quantities appear in the denomination of v(t) the spectral distribution of the noise bands A_t and B_t of Exp (8) can be obtained without the complication that associated, for instance, with a rigorous treatment of the output from a conventional F.M. receiver or indeed, the forms of receiver proposed in Refs. 1 and 3. Now M_t, A_t and B_t are mutually uncorrelated and taking B_t first, its auto-correlation function (the Fourier transform of which gives the power spectrum) may be shown (using results given in Appendix II of Ref. 4) to be:—

$$-\frac{2}{E^4}\left[g\,g''-(g')^2\right] \quad \ldots(9)$$

where

$$g=N\,.\,\frac{\operatorname{Sin}\omega_B\tau}{\omega_B\tau}$$

$$g'=\frac{\partial g}{\partial\tau}, \text{ etc.}$$

The spectral density in terms of power per radian/sec is then:—

$$\frac{\omega_B}{3}\left(\frac{N}{C}\right)^2\left(1-\frac{\omega}{2\omega_B}\right)^3 \text{ over the base-band range} \quad \ldots(10)$$

The working for A_t is more complicated and the expression for its autocorrelation function turns out to be:—

$$\left[g\left(\psi_M(\tau)+b^2\right)+2g'b-g''\right]\exp-\left[\psi_\mu(o)-\psi_\mu(\tau)\right] \quad \ldots(11)$$

Here
$\psi_M(\tau)$ is the autocorrelation function of M_t.
$\psi_\mu(t)$ is the autocorrelation function of μ_t and
$b=-\psi_\mu'(\tau)=\langle M(t+\tau)\mu(t)\rangle$ is the cross correlation between the frequency modulation and the phase modulation at two times separated by τ.

The exponential factor multiplying Exp. (11) may be recognised as being directly proportional to the autocorrelation function of the noise modulated carrier at the receiver input. The power spectrum of Exp. (11) is thus given by a convolution of the r.f. spectrum and the spectrum of

$$\left[g\left(\psi_M(\tau)+b^2\right)+2g'b-g''\right].$$

A very similar situation has been given detailed consideration in ref. (6) where it is shown that under all modulation conditions likely to be met, the exponential has little effect and can be omitted. This step is thought to be an acceptable simplification in the present situation (particularly for low deviations)

and, further, some investigation of the spectral levels over the base-band range reveals that the b^2 term makes a very minor contribution irrespective of the deviation being large or small.

Exp. (11) is therefore reduced to:—

$$g\psi_M(\tau)+2g'b-g'' \quad \ldots.(12)$$

and the spectral density of this combination is given by:—

$$3\left(\frac{N}{C}\right)\frac{\omega_\Delta^2}{2\omega_B}+\frac{1}{2\omega_B}\left(\frac{N}{C}\right)^{\omega^2} \quad \ldots.(13)$$

The demodulation performance is plotted in the form of the ratio at spot frequencies in the base-band of the spectral density of the unwanted noise to the spectral density of the band of noise simulating the frequency modulation on the incoming carrier. The abscissa scale on Figures 2 and 3 is

$$C/T \text{ dBW/°K}=10 \log_{10} (\frac{C}{N})+10 \log_{10} (kB)$$

where k=Boltzmann's Constant, T=Absolute temp. in Degrees Kelvin and B=The Noise bandwidth.

This latter quantity is given by

$$\frac{\omega_\Delta^2}{P_m},$$

and so, on collecting together Exp. (10) and Exp. (13), the distortion-to-signal ratio is given by:—

$$\frac{D}{S}=3\left(\frac{N}{C}\right)\left(\frac{P_m}{2\omega_B}\right)+\left(\frac{N}{C}\right)\left(\frac{P_m}{\omega_\Delta}\right)^2\frac{P_m}{2\omega_B}\left(\frac{\omega}{P_m}\right)^2+\frac{1}{6}\left(\frac{N}{C}\right)^2\left(\frac{P_m}{\omega_\Delta}\right)^2\left(\frac{2\omega_B}{P_m}\right)\left[1-\frac{\omega}{2\omega_B}\right]^3$$

$$=T_1+T_2+T_3$$

where 'ω' (rad/sec) is a general base-band frequency. It is to be noted that since M_t is uncorrelated with the terms A_t and B_t of Exp. (8), the behaviour common to most F.M. demodulators in that a depression of the wanted base-band signal occurs at low carrier-to-noise ratios (Refs. 4, 5, 6), is absent here.

Discussion of the Terms T_1, T_2 and T_3

The terms T_1 originates from the first two terms in Exp. (12) and may be viewed as a 'demodulation-noise' interaction brought about as a result of multiplication by the square of the envelope. It is to be particularly noticed that on forming the distortion-to-signal ratio, dependence upon both the deviation ratio and the base-band frequency is absent. Thus the usual situation in which channels situated low in the base-band are less affected than channels further up the band is not the case here nor it is possible to reduce the effects of noise by increasing the deviation when T_1 is significant.

The term T_2 derives from the third term, i.e. $-g''$, in Exp. (12) and may be recognised as the 'above threshold' expression associated with a conventional F.M. receiver, i.e. it refers to the linear portion of the D/S versus C/N characteristic when both are expressed in Decibels.

Whether or not the performance of the modified type of receiver under discussion closely agrees with that of the conventional receiver at large carrier-to-noise ratios depends upon the ratio $T_2/(T_1+T_3)$. Since T_3 varies as the square of the noise-to-carrier, however, the deciding ratio is effectively

$$T_2/T_1=\frac{1}{3}\left(\frac{P_m}{\omega_\Delta}\right)^2\left(\frac{\omega}{P_m}\right)^2$$

and for close agreement this ratio is to be large.

Finally the term T_3 is a result of the particular combination of the in-phase and quadrature noise components (and their derivatives) shown in the last bracket of Exp. (7). Here again, dependence on the base-band frequency 'ω' is very minor. T_3 becomes dominant at very low carrier-to-noise ratios and the simple dependence on

$$\left(\frac{N}{C}\right)^2$$

rather than a more complicated exponential type of variation (ref. 6) associated with more familiar types of receiver, makes it an easy matter to investigate the performance well below threshold.

It will have been noticed that the physical interpretations of the terms T_1, T_2 and T_3 have included no reference to clicks. The procedure of multiplying by the square of the envelope means that when a click is about to occur in the normal manner the envelope level is dropping and the result of the multi-

plication is presumed to be that the output signal v(t) of Exp. (6) no longer contains sharp individual peaks.

In exchange for this immunity from click-noise the output contains 'noise' and 'noise× modulation' types of components. Further, since a drop in the envelope does not necessarily imply the appearance of a click it is evident that the multiplication is introducing some undesirable modification of the demodulated signal at such times. The net effect of these distortion contributions is given by terms T_1, T_2 and T_3.

Finally, Figures 2 and 3 show a comparison of the performance of a demodulator employing the multiplication by V^2 technique with that provided by a frequency modulation feedback (FMBF) receiver. Figure 2 shows the large deviation situation and indicates that the envelope multiplication brings about no improvement over the prediction for the F.M.F.B. receiver levels derived from the theory of Ref. 6.

In contrast, when the deviation ratio is low, as in Figure 3, the demodulator of Figure 1 would appear to have attractive advantages.

There are a number of possible variations of the demodulation technique described here which might be useful in some applications such as multiplication by the envelope rather than its square, or setting a level on the output of the detector above which multiplication is by a constant.

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List of Symbols

ω_o = Carrier frequency (rad/sec).
ω_Δ = R.M.S. multi-channel frequency deviation (rad/sec).
ω = A general base-band frequency (rad/sec).
P_m = Modulation bandwidth (rad/sec).
$2\omega_B$ = Bandwidth of noise entering receiver (rad/sec).
μ_t = Phase modulation corresponding to multi-channel frequency modulation.
M_t $=\frac{d\mu_t}{dt}=$ Multi-channel frequency modulation.
E = Carrier amplitude.
b = Amplitude of a general tone in representing a band of thermal noise.
α_q = Random phase angle uniformly distributed 0—2π.
I_o = In-phase noise component.
I_s = Quadrature noise component.
V(t) = Envelope of carrier and noise combination at receiver input.
$\psi_M(\tau)$ = Autocorrelation function of M_t.
$\psi_\mu(\tau)$ = Autocorrelation function of μ_t.
C/N = Carrier-to-noise ratio at receiver input $=\frac{\frac{1}{2}E^2}{b^2\omega_B}$.

$g(\tau)$ = Autocorrelation function of I_o or I_s.

WHAT WE CLAIM IS:—

1. A frequency modulation demodulator comprising frequency discriminator means for detecting an input frequency modulated signal and amplitude detector means for detecting the amplitude of said signal, the output of the frequency discriminator means and amplitude detector means being multiplied together to afford the required output signal.

2. A demodulator as claimed in claim 1, in which the amplitude detector means takes the form of a square-law detector.

3. A demodulator as claimed in claim 1,

in which the amplitude detector means takes the form of a linear (envelope) detector.

4. A demodulator as claimed in any preceding claim, in which the input signal is fed to signal dividing means which affords signals for the amplitude detector means and the frequency discriminator means.

5. A demodulator as claimed in any preceding claim in which limiter means is provided in the input of the frequency discriminator means.

6. A demodulator as claimed in any preceding claim, comprising amplifier means for amplifying the output of the amplitude detector means.

7. A frequency modulation demodulator substantially as hereinbefore described with reference to Figure 1 of the accompanying drawings.

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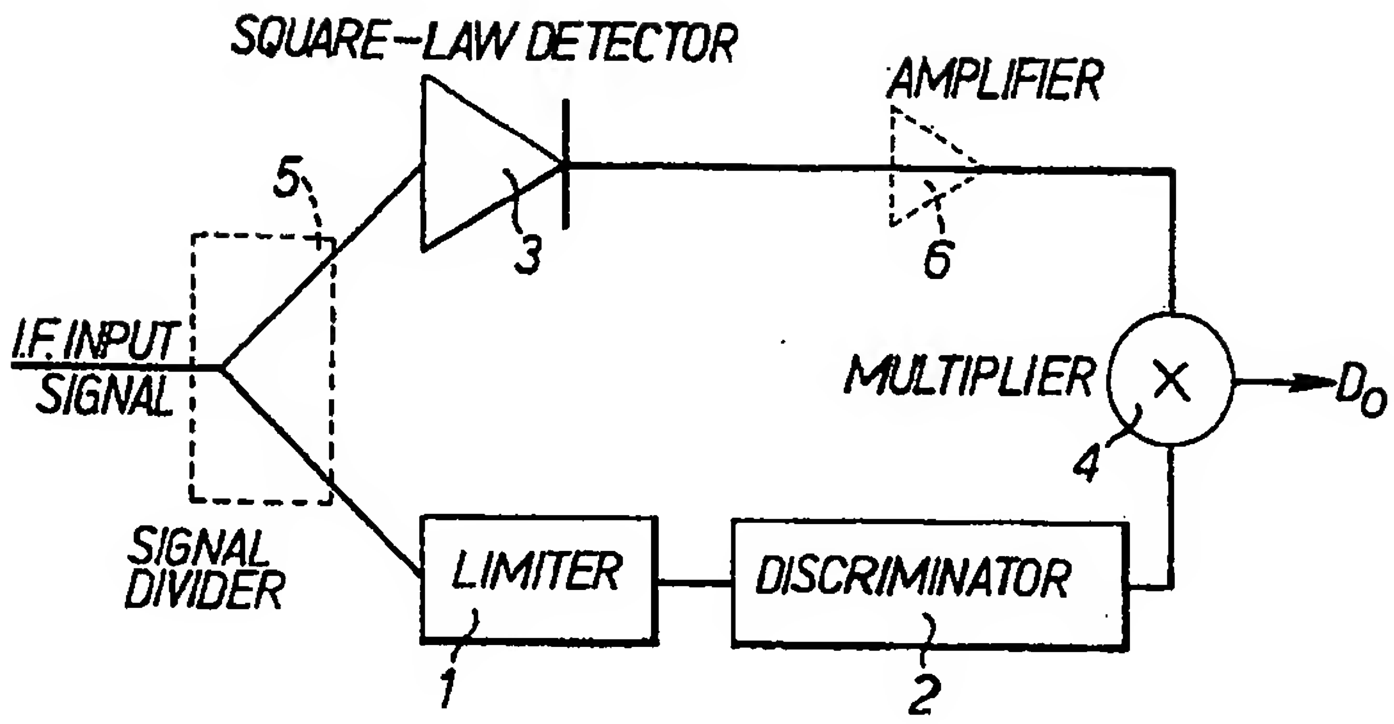
SQUARE-LAW DETECTOR
AMPLIFIER
5
3
6
I.F. INPUT SIGNAL
MULTIPLIER
D_0
4
SIGNAL DIVIDER
LIMITER
DISCRIMINATOR
1
2

FIG. 1.

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Sheet 2

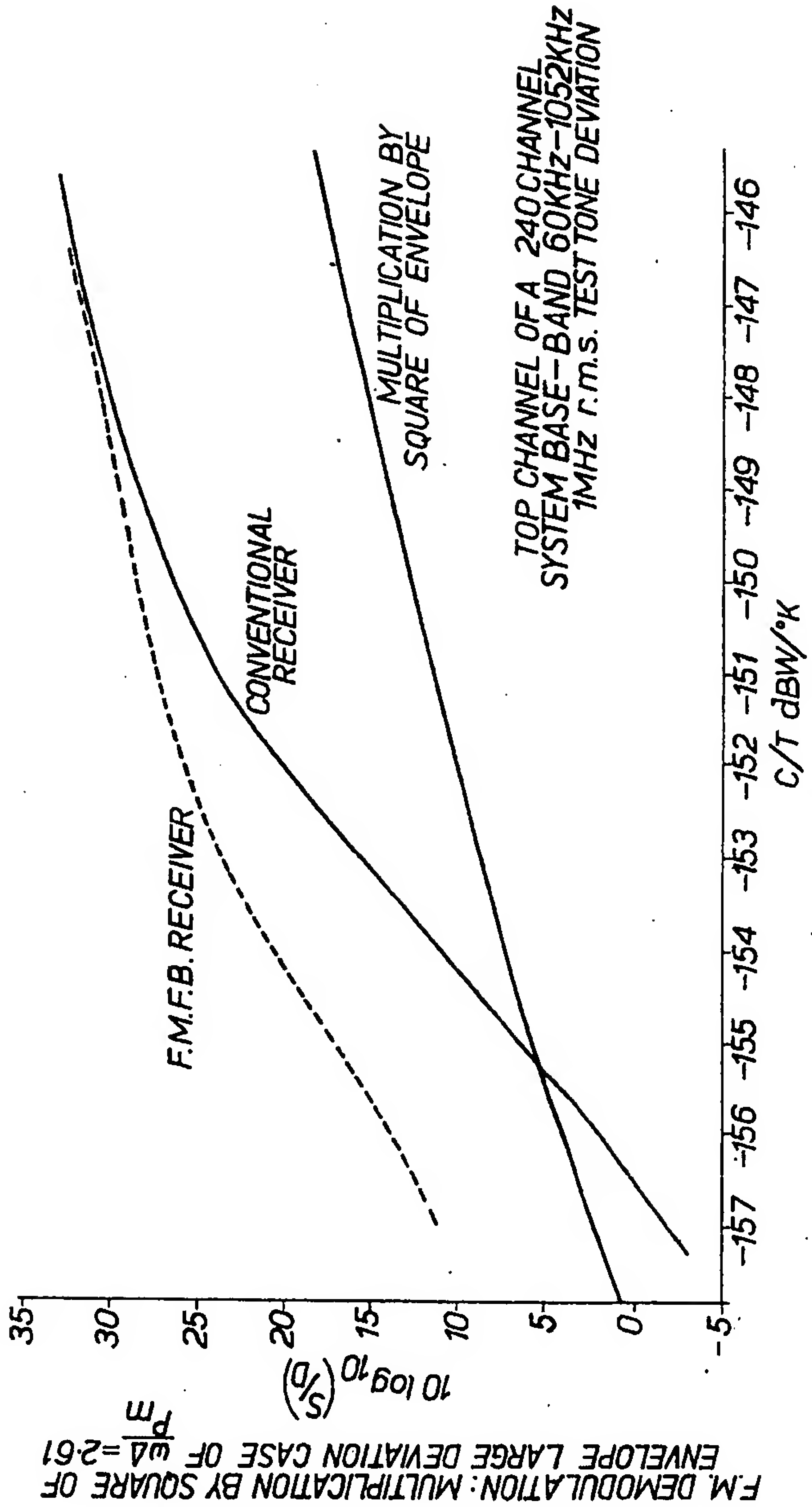

FIG. 2.

3 SHEETS

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Sheet 3

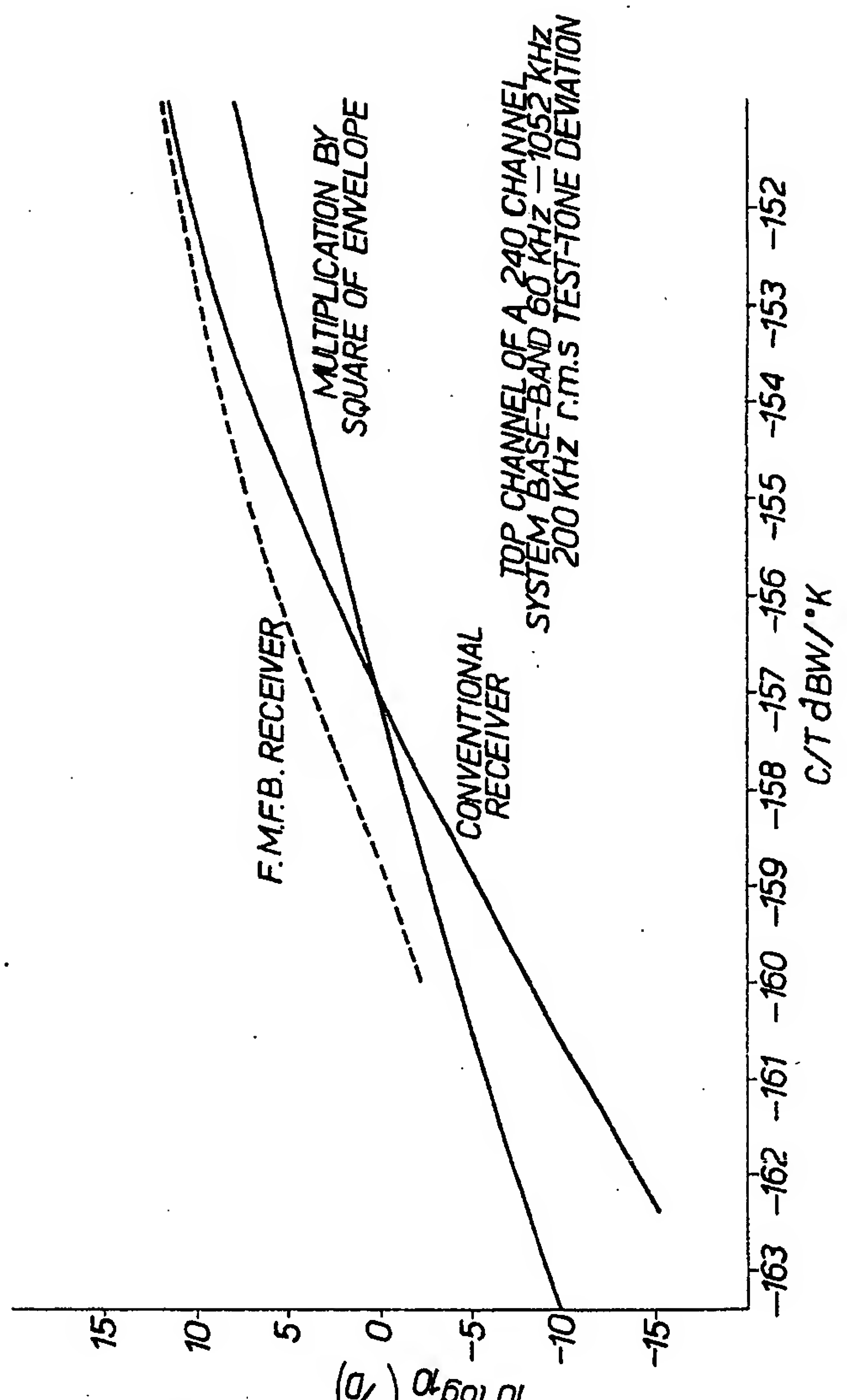

FIG. 3.

F.M. DEMODULATION : MULTIPLICATION OF SQUARE OF ENVELOPE LOW DEVIATION CASE OF $\frac{\omega_d}{p_m}=0{\cdot}52$